Sum Rate Analysis Of Massive Multiple Input Multiple Output System For Linear Precoding Using Normalization Methods

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Date: February/2016
Dedication

This work is dedicated with great thanks to my parents, my brothers and sisters …

To my relatives …

To my supervisors…

To my Examiners…

To everyone taught and will teach me a letter…

To my friends and colleagues …

…May the almighty Allah keep all of them well and healthy….
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No words can describe how grateful and indebted I am to my parents: my compassionate mother; throughout my life you are always near, A tender smile to guide my way, you are the sunshine to light my day. And my mentor father, my way, of whom I am a living will: may your happiness fulfill, your goodness, as is just and right, deeds are seeds upon the night as wind and wonder have their way, delivering the destined light. My brothers and sisters; their love and care are the motivation for all my achievements, and they will forever be. I am deeply thankful to my aunts and uncles and all my family.

Alhamdulillah, all praises to Allah firstly and lastly

‘’’ Thanks billions trillions for all of you ‘’’
Sum Rate Analysis of Massive multiple input multiple output System for Linear Precoding using Normalization Methods

Ali Mahir Ali Ibrahim

Abstract

In cellular communication networks, the massive MIMO (multiple input multiple output) system is massive number of antennas (hundreds) at the transmitters which is the most promising solution for the fifth generation of mobile communication network. In order to cope with the huge demands of communications in the network it has been found that it is useful to have multiple receivers served in the same bandwidth during the same time slot. Each receiver not only gets the waveforms which are dedicated to it but also will overhear the signals which are actually meant for the other users and this phenomena is called interference which is one of the major problem in wireless communication. The objective of this study is to mitigate the interference between the users in the same cell which is multi-user interference to enhance system performance by increasing number of base station antennas and applying linear precoding techniques. This study implemented a software code of three linear precoding techniques: zero forcing (ZF), maximum ratio transmission (MRT) and match filter (MF) for downlink massive MIMO system. The study presented a comparison of the sum rate versus total number of transmitting antenna and sum rate versus number of users for downlink massive multiple input multiple output System over perfect channel depending on vector normalization and matrix normalization methods when number of base station antennas from 1 to 200, number of user from 1 to 50 and downlink transmitting power 0dB and -15dB. In high power, the sum rate improvement for ZF vector normalization and matrix normalization are (100.389 bps/Hz, 128.530 bps/Hz) respectively and in low power (6.579 bps/Hz, 10.471 bps/Hz) respectively., for MRT vector normalization and matrix normalization are (79.973 bps/Hz, 79.732 bps/Hz) respectively and in low power, (8.347 bps/Hz, 8.387 bps/Hz) respectively, for MF vector normalization and MF matrix normalization are equal, in high power (79.973 bps/Hz) and in low power (8.387 bps/Hz).Simulations results show that using linear precoding techniques and increasing number of base station antennas enhance system performance. In conclusion ZF has better performance at vector/matrix normalization with high power. With low power, MF has better performance in vector normalization and ZF has better performance in matrix normalization. The recommendations of this study are to investigate the performance of linear precoding techniques by taking into account the imperfect channel state information (CSI) and to Study the nonlinear precoding techniques for massive MIMO system.
تحليل المعدل الجمعي لنظام متعدد المدخلات متعدد المخرجات الهائل للترميز المبديني الخطفي باستخدام طرق التوحيد

علي ماهر علي إبراهيم

ملخص الدراسة

في شبكات الاتصالات الخلوية نجد أن نظام متعدد المدخلات متعدد المخرجات الهائل هو عدد هائل من الهوائيات (مئات) في محطات الأرسال وهو الحل الواعي للجي البصري من شبهة الاتصالات المختلفة. من أجل مراقبة الضوضاءات الهائلة من الاتصالات في الشبكة فقد وجد أنه من المفيد أن يكون عدد مستقبلات تخدم في نفس النطاق التردد البصري مثل نفس الشريحة الزمنية. كل مستقبل لا ينقي فقط الإشارة المختصة إليه فحسب، ولكن أيضا سوف تصل إليه الإشارات الأخرى المخصصة للمستخدمين الأخرين، وتمس هذه الظاهرة بالداخلة التي هي واحدة من المشاكل الرئيسية في الاتصالات اللاسلكية. الهедь من هذه الدراسة هو تقليل التداخل بين المستخدمين في نفس الخلية وهي تداخل مستخدمين تتصادم أداء النظام عن طريق زيادة عدد هواويات المحطة الأساسية وتطبيق تقنيات الترميز المبديني الخطية. نفت هذه الدراسة برمجيات تتميز ثلاث تقنيات خطية للترميز المبديني نسبة الإرسال القصوى (MRT) والتصفير القسري (ZF) والمرشح المتوافق (MF) لوصول التحميل للنظام متعدد المدخلات متعدد المخرجات الهائل. قدمت الدراسة مقارنة لمعدل الربح مقابل إيجالي عدد هواويات الأرسال ومعالج الجموع مع عدد من المستخدمين لوصول التحميل لنظام متعدد المدخلات متعدد المخرجات الهائلية على قناة مثالية اعتمادا على طرق توحيد المتوجه وتوحيد المصفوفة. الدراسة فرضت عدد من هواويات المحطة الأساسية 1-020، عدد المستخدمين 1-050 وطاء الارسال لوصول التحميل 0 ديسيل و15 ديسيل (نسبة عالية وطاء متخصصة على النتائج). في حالة الطاقة العالمية (التحسن في معدل الجمع توحيد المتوجه وتوحيد المصفوفة عند الصفر القسري هو 100.0388 بت في الثانية لكل هرتز، 128.5230 بت في الثانية لكل هرتز) على النتالي وفي حالة الطاقة المتخصصة كانت (6.5799 بت في الثانية لكل هرتز، 10.4717 بت في الثانية لكل هرتز، 79.9272 بت في الثانية لكل هرتز) على النتالي. عند الطاقة العالمية هي (79.973 بيت في الثانية لكل هرتز، 84.3787 بيت في الثانية لكل هرتز، 8.3477 بيت في الثانية لكل هرتز). في حالة الطاقة المخفضة (8.3477 بيت في الثانية لكل هرتز) وتؤثر المجاورة في حالة الطاقة العالمية هي (79.7397 بيت في الثانية لكل هرتز). أظهرت النتائج أن استخدام تقنيات الترميز المبديني الخطفي يمكن تقليل عدد هواويات المحطة وتحسين أداء النظام، وفي الخلاص تقنيات الصفر القسري لديها أداء أفضل في توحيد النتائج وتقنيات المصرف القسري لديها أداء أفضل في توحيد المصفوفة. أوصت الدراسة بدراسة أداء تقنيات الترميز المبديني الخطفي بالأخذ في الاعتبار حالة قناة غير متئلة فيلا أيضا دراسة تقييمات الترميز المبديني الخطي لنظام المتعدد المدخلات المتعدد المخرجات الهائل.
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LIST OF ABBREVIATIONS

MIMO  Multiple Input Multiple Output
SU-MIMO  Single User MIMO
MUI  Multi User Interference
BS  Base Station
UE  User Equipment
LTE  Long Term Evolution
DL  Downlink
UL  Uplink
MU-MIMO  Multiuser MIMO
SD  Spatial Diversity
SM  Spatial Multiplexing
SISO  Single Input Single Output
SNR  Signal to Noise Ratio
BLAST  Bell Laboratory Layered Space-Time
SINR  Signal to Interference Noise Ratio
SIMO  Single Input Multiple Output
MISO  Multiple Input Single Output
LTE-A  LTE-Advanced
MT  Mobile Terminal
GSM  Global System for Mobile
CSI  Channel State Information
DPC  Dirty Paper Coding
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<td>UTs</td>
<td>User Terminals</td>
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<td>DU</td>
<td>Digital Unit</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximum Ratio Combining</td>
</tr>
<tr>
<td>A2SE</td>
<td>Average Area Spectrum Efficiency</td>
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<td>EE</td>
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CHAPTER ONE
INTRODUCTION

1.1 Overview

In previous years, having to go to a fixed location in order to make a telephone call was an inconvenience. Nowadays this inconvenience is minimized because the facility of a mobile phone allows someone to carry out this mundane task almost whenever or whenever they please. As mobile communications have grown exponentially in recent years and, in parallel, the World Wide Web and its applications have spread widely, the possibility to have access to Internet, entertainment and multimedia communications wirelessly has accelerated a similar trend: people want to avoid further inconveniences of having to go to a fixed location and establish the necessary communication link to get access to the Internet. Moreover, it is desirable to avoid having to install the corresponding cables into a building or home and to avoid incurring the related costs [1].

The bandwidth of wireless communication systems is often limited by the price of the radio spectrum required and the complicated wireless propagation environment. Any increase in data rate, which can be realized without increasing the bandwidth, but with a reduction in power consumption, makes the system more spectrally and power efficient and less costly [2].

Multiple inputs multiple output (MIMO) systems have emerged as an enabling technology to achieve the design goals of contemporary communication systems and have given rise to a proliferation of research activity worldwide. MIMO, technology relies on multiple antennas to simultaneously transmit multiple streams of data in wireless communication systems. When MIMO is used to communicate with several terminals at the same time, we speak of multiuser MIMO, MU-MIMO for short. Massive MIMO is an emerging technology that scales up MIMO by possibly orders of magnitude compared to current state-of-the-art. With massive MIMO, we consider of systems that use antenna arrays with a few hundred antennas, simultaneously serving many tens of terminals in the same time-frequency resource. The basic premise behind massive MIMO is to gain all the benefits of conventional MIMO, but on a much greater scale. Overall, massive MIMO is an enabler for the development of future broadband (fixed and mobile)
networks which will be energy-efficient, secure, and robust, and will use the spectrum efficiently [3]. Linear precoding methods usually achieve rational performance with much lower complexity. Linear precoding strategies include maximum ratio transmission (MRT) precoding, zero-forcing (ZF) precoding and Match Filter (MF) precoding. Which clues to enhance the system quality of service by reduce the impact of multiuser interference with assumption perfect channel state information.

1.2 Problem Statement

The main problem faced by wireless communication systems is interference. In downlink channel (broadcast channel) for massive MIMO system, the major impact factor is Multi-user interference. MUI is based on the interference from other user in the same cell on the channel state information, which led to reduce quality of service for mobile communication.

1.3 Objectives

- The specific goals of this dissertation is to analyze and investigate the performance of three linear precoding techniques (Zero Forcing (ZF), Maximum Ratio Transmission (MRT) and Match Filter (MF)) to mitigate or eliminate the effect of Multi user Interference (MUI) for single cell downlink massive MIMO system at perfect channel.

- Compare the performance of Zero Forcing (ZF), Maximum Ratio Transmission (MRT) and Match Filter (MF) by a comparison of the sum rate versus total transmitting antenna and sum rate versus number of users at high and low power.

1.4 Methodology

Implementing software MATLAB code of ZF, MRT and MF precoding techniques. The study proposed four different scenario of calculating the sum rate of each precoding technique in a single-cell downlink massive MIMO system at different value of power under the same assumptions (perfect channel state information), the first and second scenarios accomplished by taking different numbers of antennas, the other scenarios assumed a different number of users. All this is done with respect to the normalization methods. Finally a comparative analysis of
results for the two scenarios of normalization methods presented in term of sum rate overall precoding techniques.

A. Zero-forcing (ZF) precoding

ZF is one technique of linear precoding in which the inter user interference can be cancelled out at each user.

B. Maximum ratio transmission (MRT) precoding

MRT is one technique of linear precoding which maximizes the signal gain at the intended user.

C. Matched Filter (MF) precoding

MF is one technique of linear precoding which mitigate the interference for multi-user by matching filters for channels which means that the coefficient of the weights factor of the weight vector are equal to the conjugate complex entries of the channel coefficients.

The performance of three precoding schemes in terms of sum rate in a single-cell downlink, because the signal-to-interference-noise ratio is a function of the transmit beamforming vector, and ZF, MRT and MF have different beamforming vectors according to concepts of three techniques.

1.4 Thesis Contributions

This dissertation contribute by investigate the sum rate analysis for three linear precoding techniques: zero forcing, maximum ratio transmission and match filter and compare between them according to different number of antennas and different number of users with high and low power to find the optimal technique among other precoding techniques.

1.5 Thesis Outline

Thesis consist five chapters. Chapter one is an introduction. Chapter two presents literature review for massive MIMO system. The third chapter illustrates the methodology. Chapter four introduce the simulation and results, lastly chapter five gives the conclusion and recommendations for future works.
CHAPTER TWO
BACKGROUND AND LITERATURE REVIEW

2.1 Background
During the last years, data traffic (both mobile and fixed) has grown exponentially due to the
dramatic growth of smartphones, tablets, laptops, and many other wireless data consuming
devices. The demand for wireless data traffic will be even more in future [4-7].
In wireless communication, the transmitted signals are being attenuated by fading due to
multipath propagation and by shadowing due to large obstacles between the transmitter and the
receiver and by the interference between the users at the same cell due to huge demand to gain
high data rates to every single user, yielding a fundamental challenge for reliable
communication.
Furthermore, with multiple antennas, multiple streams can be sent out and hence, we can obtain a
multiplexing gain which significantly improves the communication capacity. MIMO systems
have gained significant attention for the past decades, and are now being incorporated into
several new generation wireless standards (e.g., LTE-Advanced, 802.16m).
The effort to exploit the spatial multiplexing gain has been shifted from MIMO to multiuser
MIMO (MU-MIMO), where several users are simultaneously served by a multiple-antenna base
station (BS). With MU-MIMO setups, a spatial multiplexing gain can be achieved even if each
user has a single antenna [4]. This is important since users cannot support many antennas due to
the small physical size and low cost requirements of the terminals, whereas the BS can support
many antennas.
MU-MIMO does not only reap all benefits of MIMO systems, but also overcomes most of
propagation limitations in MIMO such as ill-behaved channels. Specifically, by using scheduling
schemes, we can reduce the limitations of ill-behaved channels.
Line-of-sight propagation, which causes significant reduction of the performance of MIMO
systems, is no longer a problem in MU-MIMO systems. Thus, MU-MIMO has attracted
substantial interest [8-13].
There always exists a tradeoff between the system performance and the implementation
complexity. The advantages of MU-MIMO come at a price [4]:

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• Multiuser interference: the performance of a given user may significantly degrade due to the interference from other users. To tackle this problem, interference reduction or cancellation techniques should be used. These techniques are complicated and have high computational complexity.

• Acquisition of channel state information: in order to achieve a high spatial multiplexing gain, the BS needs to process the received signals coherently. This requires accurate and timely acquisition of channel state information (CSI). This can be challenging, especially in high mobility scenarios.

• User scheduling: since several users are served on the same time-frequency resource, scheduling schemes which optimally select the group of users depending on the precoding/detection schemes, CSI knowledge etc., should be considered. This increases the cost of the system implementation. The more antennas the BS is equipped with, the more degrees of freedom are offered and hence, more users can simultaneously communicate in the same time-frequency resource.

Multiple inputs multiple outputs which here refer for to the deployment of multiple antenna elements at the transmitters and receivers of communications links. MIMO enhances the wireless communications link with an extra dimensional in space, this means that the advantages that comes with MIMO are very similar to the benefit that we have with two ears instead of only one ear which allows us to distinguish Sounds from different directions. For the technological equivalent for MIMO system we can say this means that a technique allow to exploit the directivity of signals for an extra dimension or recourse on top of bandwidth for the wireless communication link [14].

This is the reason why MIMO has become an enabling technology for the current fourth generation of mobile communication better known as LTE and LTE advance, for future generation of mobile communication systems - the next one is the fifth generation - we are still seeking solution for keeping pace with the ever increasing demand for higher data rates in more connected world of even today and more tomorrow.

One of the most promising solutions for these challenges is massive MIMO which is MIMO but with a massive number of antenna - hundreds to thousands – at the transmitters. Obviously this will change the shape of future antennas traumatically. You might imagine that the whole
fasciate of a building becomes a huge antenna array with multiple of adaptive integrated antennas in fig.2.1

Fig.2.1 whole fasciate of a building becomes a huge antenna array

Massive MIMO is definitely not just a scaling up of standard MIMO on the contrary, in order to obtain the potentials and benefits of the massive MIMO we have to reinvent and rethink lots of the algorithms and concepts from standard MIMO and that poses a variety of very interesting and challenging research questions for us researches in academia as well as industry [14].

2.1.1 Channel systems model

Let’s first start with the physical wireless communication link the so called SISO link as shown in the following fig.2.2
Fig. 2.2 SISO system

Here is one single antenna element at the transmitter which is radiating electromagnetic waves fronts to the receiver in the right hand side. The main issue we see, wireless medium is the attenuations of the signal strength so if the signal has to travel a couple of hundred meters or even kilometers from the transmitter to the receiver, the signal experiences a huge attenuation this is about effect of one hundred million which refers to attenuation of -80 dB, that means that the signal at the receiver is extremely weak as shown in fig.2.3. One of the solutions to this problem is the deployment of multiple antennas at the transmitter which in MISO (multiple input single output) in order to study this in more details, we will replace this more abstract illustration as fig.2.4 by the notation as shown in fig.2.5.

Fig. 2.3 Attenuation in SISO
Here each antenna at the transmitter is linked to the antenna at the receiver and characterized by a complex number $h_1, h_2, h_3, h_4$, which relates to a so-called based band representation model which also allow us to describe the signal at the receiver ($y$) as a linear combination of the transmitted signals $x_1, x_2, x_3, x_4$ which arrives at the receiver with linear weights $h_1, h_2, h_3, h_4$ which equals the respective channel coefficients, this is also can be expressed in a more complicit version by an inner product of a channel vector $h$ with the transmit vector $x$. And in all this models we have of course an additional noise term ($n$) that refers to any kind of temporal or any other kind of noise at the receivers as shown in fig.2.6.
Fig.2.6  Mathematical model of MISO

The question is how to design this transmit signals as in fig.2.7 if we want to transmit a dedicated signal s to the receiver in the right hand side, and this is done by one of the most prominent solutions MF, MRT and ZF.

As example, the match filter means that the coefficient of the weights factor $w_1$, $w_2$, $w_3$ and $w_4$ of the weight vector are equal to the conjugate complex entries of the channel coefficients. And this has the effect that we are able to concentrate or focus the radiated wave forms in the direction of the receiver and this means that there is no energy wasted and no power wasted into directions where no receiver is. The respective receive signal then $y$ is equal to inner product of the channel vector $h$ with its conjugate complex copy normalized by it’s for the strength of the vector this basically means that the received signal is the dedicated signal $s$ with a scaled version of vector $h$ which is equal to the length of the channel vector the norm of the channel vector, and then we have the respective noise as illustrated bellow in fig.2.8.
For the relation of the signal portions of the SNR which is ratio between powers of the desired signal over the power of the noise and this show us that under certain requirements and assumptions this proportional to $M$ which is the number of antenna elements and this called the Array gain [14].

Multiuser communication, because communication in a cellular network is not a single show between one base station and one user because all of us want to have cellular communication services by the providers, in order to cope with the huge demands of communications in the network it is has been found that is useful to have multiple receivers served in the same bandwidth during the same time slot and what the problem is can be seen from the following illustration in fig.2.9.

Each receiver not only gets the waveforms which is dedicated to it but also will overhear the signals which actually meant for the other users and this phenomena is called interference which is the second major problem in wireless communication.
To study this in more details we replace the illustration by more mathematical one as shown in fig.2.10. For each link from the base station to the receivers is represented here by a channel vector, and each channel vector consist of so many channel coefficients as we have transmit antennas. For the signal model that means for example at receiver $i$, $y_i$ is the inner product of the respective channel vector $h_i$ with the compound signal vector $x_i$ plus a noise term.

![Mathematical model for MIMO](image)

**Fig.2.10  Mathematical model for MIMO**

Of all dedicated signal to all users, because all users share the same medium (same bandwidth and time slot) so the compound signal is a linear combination of all signals with their respective beamformer (MF, MRT, ZF). The table 2.1 below illustrates the various antenna types:

<table>
<thead>
<tr>
<th>Multi-antenna types</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SISO</td>
<td>Single-input-single-output means that the transmitter and receiver of the radio system have only one antenna.</td>
</tr>
<tr>
<td>SIMO</td>
<td>Single-input-multiple-output means that the receiver has multiple antennas while the transmitter has one antenna.</td>
</tr>
<tr>
<td>MISO</td>
<td>Multiple-input-single-output means that the transmitter has multiple antennas while the receiver has one antenna.</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-input-multiple-output means that the both the transmitter and receiver have multiple antennas.</td>
</tr>
</tbody>
</table>

**Table 2.1 illustrates of various antennas types.**
The signal now at the receiver which is an x at the transmitter model consist of these three terms the first one which have already been introduced in the single user model: the dedicated signal to user $s_i$ time the norm of its channel vector, and then we have the noise term which also have been mentioned in the single user case, but now we have signals that comes from overhearing of the other users and this are interference. The interference depends on the inner product of the channel user $h_i$ with the channel vectors of all the other users, so if this term does not vanish which is typically the case that interference at the worst case may even be larger than the useful signal and this obviously will corrupt the signal transmission and information transmission and can even break down the communication link as shown in equation.2.1.

$$y_i = \|h_i\|s_i + \sum_{j=1}^{k} \frac{h_i^T h_j^*}{\|h_j\|} s_j + n_i$$  \hspace{1cm} (2.1)

In term of signal to interference and noise this means that we have here an enhanced version of the signal noise which is so called signal to interference and noise ratio that means in the denominator we have an extra term which come from the interference and this is a major problem in multiuser communication systems as shown in equation.2.2 , and one of the typical solution is either equalization at the receivers or pre-equalization at the transmitters , which means that we will choose a pre-coding or pre-equalization at the transmitter to take into account this phenomena .

$$SNR_i = \frac{\|h_i\|^2 \sigma^2_{s_i}}{\sigma^2_{n_i}} \rightarrow SINR_i = \frac{\|h_i\|^2 \sigma^2_{s_i}}{\sum_{j=1}^{k} \frac{h_i^T h_j^*}{\|h_j\|} \sigma^2_{s_j} + \sigma^2_{n_i}}$$ \hspace{1cm} (2.2)
Multiuser MIMO techniques and performance have begun to be intensely investigated because of several key advantages over single user MIMO communications:

- **MU-MIMO schemes** allow for a direct gain in multiple access capacity (proportional to the number of base station (BS) antennas) thanks to so-called multiuser multiplexing schemes.
- **MU-MIMO** appears more immune to most of propagation limitations plaguing single user MIMO communications such as channel rank loss or antenna correlation. Although increased correlation still affects per-user diversity, this may not be a major issue if multiuser diversity [15] can be extracted by the scheduler instead. Additionally, line of sight propagation, which causes severe degradation in single user spatial multiplexing schemes, is no longer a problem in multiuser setting.

MU-MIMO allows the spatial multiplexing gain at the base station to be obtained without the need for multiple antenna terminals, thereby allowing the development of small and cheap terminals while intelligence and cost is kept on the infrastructure side.

MU-MIMO in cellular communication systems brings improvements on four fronts [3]:

- **Increased data rate**, because the more antennas, the more independent data streams can be sent out and the more terminals can be served simultaneously.
- **Enhance reliability**, because the more antennas the more distinct paths that the radio signals can propagate over.
- **improved energy efficiency**, because the base station can focus its emitted energy into the spatial directions where it knows that the terminals are located.
- **Reduced interference**, because the base station can purposely avoid transmitting into directions where spreading interference would be harmful.

**Massive MIMO** (also known as “Large-Scale Antenna Systems”, “Very Large MIMO”, “Hyper MIMO”, “Full-Dimension MIMO” and “ARGOS”) makes a clean break with current practice through the use of a large excess of service-antennas over active terminals and time division duplex operation. Extra antennas as shown in fig.2.11 help by focusing energy into ever-smaller regions of space to bring huge improvements in throughput and radiated energy efficiency.
Fig. 2.1 Massive MIMO system

Other benefits of massive MIMO include the extensive use of inexpensive low-power components, reduced latency, simplification of the media access control (MAC) layer, and robustness to intentional jamming. The anticipated throughput depend on the propagation environment providing asymptotically orthogonal channels to the terminals, but so far experiments have not disclosed any limitations in this regard. While massive MIMO renders many traditional research problems irrelevant, it uncovers entirely new problems that urgently need attention: the challenge of making many low-cost low-precision components that work effectively together, acquisition and synchronization for newly-joined terminals, the exploitation of extra degrees of freedom provided by the excess of service-antennas, reducing internal power consumption to achieve total energy efficiency reductions, and finding new deployment scenarios[3], in this thesis we focusing on multi-user interference.

2.1.2 The importance of Massive MIMO

Massive MIMO technology relies on phase-coherent but computationally very simple processing of signals from all the antennas at the base station. The essential characteristics of massive MIMO are many physical small, low power antennas, aggressive spatial multiplexing and utilize measured channel characteristics.
In Massive MIMO, it is expected that each antenna would be contained in an inexpensive module with simple processing and a low-power amplifier. The main benefits of Massive MIMO systems or it called specific benefits of a massive MU-MIMO system are:

In general they are Scalability, spectral efficiency, simplicity, great service to all users and energy efficiency. In details [16]

- **Huge spectral efficiency and high communication reliability:** Massive MIMO inherits all gains from conventional MU-MIMO, i.e., with $M$-antenna BS and $K$ single-antenna users, we can achieve a diversity of order $M$ and a multiplexing gain of $\min(M, K)$. By increasing both $M$ and $K$, we can obtain a huge spectral efficiency and very high communication reliability.

- **High energy efficiency:** In the uplink Massive MIMO, coherent combining can achieve a very high array gain which allows for substantial reduction in the transmit power of each user. In the downlink, the BS can focus the energy into the spatial directions where the terminals are located. As a result, with massive antenna arrays, the radiated power can be reduced by an order of magnitude, or more, and hence, we can obtain high energy efficiency. For a fixed number of users, by doubling the number of BS antennas, while reducing the transmit power by two, we can maintain the original the spectral efficiency, and hence, the radiated energy efficiency is doubled.

- **Simple signal processing:** For most propagation environments, the use of an excessive number of BS antennas over the number of users yields favorable propagation where the channel vectors between the users and the BS. Introduction pair wisely (nearly) orthogonal. Under favorable propagation, the effect of inter user interference and noise can be eliminated with simple linear signal processing (liner precoding in the downlink and linear decoding in the uplink).

- **Massive MIMO enables a significant reduction of latency** on the air interface; The performance of wireless communications systems is normally limited by fading. The fading can render the received signal strength very small at some times. This happens when the signal sent from a base station travels through multiple paths before it reaches the terminal, and the waves resulting from these multiple paths interfere destructively. It is this fading that makes it hard to build low-latency wireless links. If the terminal is
trapped in a fading dip, it has to wait until the propagation channel has sufficiently changed until any data can be received. Massive MIMO relies on the law of large numbers and beamforming in order to avoid fading dips, so that fading no longer limits latency.

- Massive MIMO can increase the capacity and simultaneously, improve the radiated energy-efficiency. The capacity increase results from the aggressive spatial multiplexing used in massive MIMO.

- Massive MIMO can be built with inexpensive, low-power components; Massive MIMO is a game-changing technology both with regard to theory, systems and implementation. With massive MIMO, expensive, ultra-linear 50 Watt amplifiers used in conventional systems are replaced by hundreds of low-cost amplifiers with output power in the milli-Watt range.

- Massive MIMO reduces the constraints on accuracy and linearity of each individual amplifier and RF chain. All what matters is their combined action. In a way, massive MIMO relies on the law of large numbers to make sure that noise, fading and hardware imperfections average out when signals from a large number of antennas are combined in the air together. The same property that makes massive MIMO resilient against fading also makes the technology extremely robust to failure of one or a few of the antenna units.

- Massive MIMO simplifies the multiple-access layer; Owing to the law of large numbers, the channel hardens so that frequency-domain scheduling no longer pays off. With OFDM, each subcarrier in a massive MIMO system will have substantially the same channel gain. Each terminal can be given the whole bandwidth, which renders most of the physical-layer control signalling redundant.

The scientific basis for massive MIMO is about using measured channel measurements, beamforming gain grows linearly with number of antennas, irrespective of the noisiness of the measurements. Frequency-independent power control, based solely on long-scale (slow) fading, is exceedingly effective [17].
As a result, simple linear processing schemes are nearly optimal. Another key property of Massive MIMO is channel hardening. Under some conditions, when the number of BS antennas is large, the channel becomes (nearly) deterministic, and hence, the effect of small-scale fading is averaged out. The system scheduling, power control, etc., can be done over the large-scale fading time scale instead of over the small-scale fading time scale. This simplifies the signal processing significantly.

### 2.1.3 Precoding techniques

There are various precoding schemes, each of which has been designed to meet a certain criterion. Depending on how the transmitted signals are related to the input data streams, the precoding techniques are categorized as linear and nonlinear. For example [16], it has been shown that the dirty paper coding (DPC), which is a nonlinear precoding, is capable of achieving the downlink capacity [18]. Nevertheless, due to its very high complexity, some less complex nonlinear precoders like vector perturbation [19–21] and Tomlinson-Harashima [22, 23] are also of particular interest. The simplest transmission scheme for multi-antenna downlink is channel inversion [24], which is linear, such that the intra-cell interferences are pre-cancelled at BS in order to enable each MT to receive its intended signal interference free. Zero-forcing (ZF) precoding is one technique of linear precoding in which the inter-user interference can be cancelled out at each user [25]. Maximum ratio transmission (MRT) precoding MRT is one technique of linear precoding which maximizes the signal gain at the intended user [25]. Block diagonalization (BD) is a linear pre-coding technique for the downlink of MU MIMO systems [26]. It decomposes a MUMIMO downlink channel into multiple parallel independent single-user MIMO downlink channels. Another technique also proposed in [26], named successive optimization (SO), addresses the power minimization and the near-far problem and it can yield better results in some situations but its performance depends on the power allocation and the order in which the users’ signals are pre-processed. The zero MUI constraint is relaxed and a certain amount of interference is allowed.

A brief comparison between linear and nonlinear precoding techniques: Linear precoding techniques often have lower computational complexity than nonlinear ones. However nonlinear techniques often come with the advantage of improved performance by high computational complexity as shown in fig.2.12
Fig. 2.13  Relation between linear and non-linear precoding techniques

2.2 Literature review and related works

The technology massive MIMO has considered the interest of many scholars with much emphasis on spectral efficiency, power efficiency, signal processing and interference for cellular communication systems. Linear precoding schemes are an important factor in massive MIMO signal processing. The following paragraphs present the previous studies in receive diversity techniques for MIMO systems.

The use of an excessive number of BS antennas compared with the number of active users makes simple linear processing nearly optimal [27].

Network massive multiple input multiple-output (MIMO) systems have been proposed, where three radio units (RUs) connected via one digital unit (DU) support multiple user equipment (UEs) at a cell-boundary through the same radio resource, i.e., the same frequency/time band. For precoding designs, zero-forcing (ZF) and matched filter (MF) with vector or matrix normalization are considered. We also derive the formulae of the lower and upper bounds of the achievable sum rate for each precoding. Based on our analytical results, we observe that vector normalization is better for ZF while matrix normalization is better for MF. Given antenna configurations, we also derive the optimal switching point as a function of the number of active users in a network. Numerical simulations confirm our analytical results [28].

The downlinks of a time-division duplexing Multi-cell multiuser MIMO system are considered where the base stations (BSs) are equipped with a very large number of antennas. they derive the approximations of achievable rates with linear precoding techniques are derived, namely Eigen-beamforming (BF) and regularized zero-forcing (RZF). The result shows that a simple RZF
precoding scheme can achieve the same performance as BF with one order of magnitude fewer antennas in both uncorrelated and correlated fading channels [29].

The performance of such precoding schemes are analysed in terms of spectral efficiency in a single-cell downlink scenario, fixed the same value of signal-to-interference-to-noise ratio for both precoders; which should not be done for a good analysis [25].

The trade-off between the energy efficiency (as measured in bits/J) and spectral efficiency (as measured in bits/channel use/terminal) are given. It is shown that the use of moderately large antenna arrays can improve the spectral and energy efficiency with orders of magnitude compared to a single-antenna system [30].

The overview of massive MIMO system for next generation is given [3].

The performances of zero-forcing (ZF) and maximum ratio transmission (MRT) are analysed and compared in a downlink massive multiple-input multiple output system. Simulation results are found to coincide with the theoretical results, and show that ZF performs better than MRT under the same conditions [31].

MIMO systems are considered for both downlink and uplink scenarios, where three radio units (RUs) connected via one digital unit (DU) support multiple user equipment (UEs) at the cell-boundary through the same radio resource, i.e., the same time-frequency slot. Zero-forcing (ZF) and maximum ratio transmission (MRT) are considered as downlink transmitter options, while ZF and maximum ratio combining (MRC) are considered as uplink receiver options. They derive simple closed form formulas for the sum rate of each such technique [32].

To further maximize the network capacity, several network MIMO algorithms with multiple receive antennas have been proposed [33]. These systems assume, however, that the network supports maximums of three users through a relatively small number of transmit antennas.

The assumption of an infinite number of antennas at the transmitter, however, is not, in practice, really feasible [34].

The impact of pilot contamination are showed theoretically and numerically, proposing a multi-cell minimum mean square error (MMSE) based precoding algorithm to reduce both intra- and inter-cell interference[35]. The importance of resource allocation for massive MIMO was described where initial guidelines were given [36].

The problem of designing transmits beamformer and power for downlink cooperative base-station (BS) system with a large antenna arrays are taken. Two scenarios according to the power
constraints of cooperative BSs are considered. The solution strategies for both scenarios are proposed: For the sum power constraint case, a simple intuitive solution, where the power is allocated without regard to the power constraint until the SINR constraints is satisfied, is presented. For the per BS power constraints case, they use the properties of a large antenna arrays to find the solution of closed form. The performance of proposed strategy is convergent to the optimal performance are demonstrated, via numerical simulation which is achieved by using the iterative algorithm [37].

The performance for the downlink massive MIMO system is investigated when the base station serves multiple user terminals (UTs) using zero-forcing precoding. Regarding the lower bound as the approximate A2SE, the optimal number of UTs maximizing the A2SE is attained for given the number of transmission antennas and the equivalent transmission signal-to-noise ratio. The trade-off between energy efficiency (EE) and A2SE is established, and the optimal EE with respect to A2SE is deduced. Simulation results coincide to the analysis well, and they indicate that deploying more transmission antennas or multiplexing a rational number of UTs can improve the A2SE and EE, increasing the degrees of freedom will better both the outage probability and bit error ratio [38].

In spite of the above works is provided results about the performances of the linear precoding schemes, they did not give the comparison of ZF, MRT and MF performances in terms of sum rate versus number of transmit antennas and number of users in a single cell downlink massive MIMO systems.

This research analyses and compares the performance of ZF, MRT and MF; in terms of sum rate versus number of transmit antenna and sum rate versus number of users in a single-cell downlink massive MIMO system with high and low power.
CHAPTER THREE

METHODOLOGY

This chapter demonstrate the idea of using zero forcing, maximum ratio transmission and Match filter linear precoding techniques for a single cell downlink massive MIMO system to assess the system performance (Achievable Sum Rate) using the vector and matrix normalization methods under the assumption of base station with perfect channel state information and channel matrix are modelled as independent complex Gaussian random variables with zero mean and unit variance. Consider the single-cell downlink massive MIMO system shown in Fig: (3.1). The base station equipped with large number of M antennas and each k user equipped with one single antenna ($M \gg K$).

![Figure 3.1 single-cell downlink massive MIMO](image)

3.1 Mathematical model

In a single-cell downlink massive MIMO system, the system model shown in Fig: (3.2). In this thesis; does not consider pilot contamination and assume perfect channel state information at the base station (BS).
Fig. 3.2 system model for massive mem

Where

- $N_{tx}$: Number of transmission antenna
- $N_{rx}$: Number of receive antenna
- $K$: Number of users
- $W$: Weight matrix $(N_{tx} \times N_{rx})$
- $H$: Channel matrix $(N_{rx, k} \times N_{rx})$

- $w_k$: The column vector of transmit precoding
- $s_k$: The transmit symbol for the $k$th user at downlink

A channel matrix is modeled as independent complex Gaussian random variables with zero mean and unit variance. Then, the receiver vector is given by equation (3.1):

$$y = \sqrt{P_d} H x + n = \sqrt{P_d}HWx + n \quad \cdots \quad (3.1)$$

The received signal at the $k$th user is expressed by:

$$y_k = \sqrt{P_d} h_{rk}^T w_k s_k + \sqrt{P_d} \sum_{i \neq k} h_{rk}^T w_i s_i + n_k \quad \cdots \quad (3.2)$$
Where

\( P_d \): Downlink transmits power.

The signal now at the receiver which is an X at the transmitter model consist of these three terms the first one which have already been introduced in the single user model: the dedicated signal to user si time the norm of its channel vector, and then we have the noise term which also have been mentioned in the single user case, but now we have signals that comes from overhearing of the other users and this are interference.

The signal-to-interference-plus-noise ratio of the \( k \)th user is given by

\[
\text{SINR}_k = \frac{P_d |h_w w_k|^2}{P_d \sum_{i=1}^{k} |h_w w_k|^2 + 1}
\]  

\( ... \)  (3.3)

In term of signal to interference noise this means that we have here an enhanced version of the signal noise which is so called signal to interference ratio that means that in the denominator we have an extra term which come from the interference and this is a major problem in multiuser communication systems , and one of the typical solution is either equalization at the receivers or pre-equalization at the transmitters , which means that we will choose a pre-coding or pre-equalization at the transmitter to take into account this phenomena.

3.2 Linear precoding (pre equalizing) techniques

The major problem here is multi-user interference, so we can cancel it or technically eliminate this interference signal we can use the linear precoding techniques (Zero forcing precoding (ZF), Maximum ratio transmission precoding (MRT) and match filter precoding.

3.2.1 Zero forcing precoding (ZF) technique

ZF precoding is one technique of linear precoding in which the multiuser interference can be mitigated at each user. This precoding is assumed to implement a pseudo-inverse of the channel matrix. The ZF precoding is given by equation (3.4):

\[
W_{ZF} = H^H (HH^H)^{-1} = [w_1 w_2 \ldots w_k]
\]  

\( ... \)  (3.4)
Where: $W$ is a precoding matrix consisting of each column vector $w_k$ of the $k^{th}$ user

$H$: channel matrix

### 3.2.2 Maximum ratio transmission precoding (MRT) technique

MRT is one technique of linear precoding where the signal to noise ratio at the each user can be maximized by implementing the hermitian of channel matrix (conjugate transpose of channel matrix). The MRT precoding given by

$$W_{MRT} = H^H = [w_1 w_2 \ldots w_k] \ldots \quad (3.5)$$

Where: $W$ is a precoding matrix consisting of each column vector $w_k$.

### 3.2.3 Matched Filter precoding (MF) technique

MF is one technique of linear precoding which mitigate the interference for multi-user by matching filters concept. The MF precoding employed by the BS is written as:

$$W_{MF} = H^* = [w_1 w_2 \ldots w_k] \ldots \quad (3.6)$$

$H^*$: conjugate transpose of channel matrix

### 3.3 Normalization method

The precoding matrix is normalized to comply with the power constraint using two normalization methods

- Matrix normalization with
  $$w_k = w_1/\sqrt{\|w_k\|} \ldots \quad (3.7)$$

- Vector normalizations
  $$w_k = w_k/\|W\|_w \ldots \quad (3.8)$$

In this thesis, for simplicity, we neglect the power optimization that could result in a complexity in massive MIMO antenna systems. Knowing that equal power per downlink stream is caused by vector normalization, while matrix normalization imposes streams with different power.
3.3.1 MF/ MRT /ZF with vector normalization

The received signal at the k_{th} user can be expressed as:

\[ y_k = \sqrt{P_d} h_k^T \frac{w_k}{\sqrt{K} \|w_k\|} s_k + \sqrt{P_d} \sum_{l=1 \atop l \neq k}^{K} h_l^T \frac{w_l}{\sqrt{K} \|w_l\|} s_l + n_k \quad \ldots \quad (3.9) \]

3.3.2 MF/ MRT /ZF with matrix normalization

The received signal can be change with matrix normalization as

\[ y_k = \sqrt{P_d} h_k^T \frac{w_k}{\|W\|_w} s_k + \sqrt{P_d} \sum_{l=1 \atop l \neq k}^{K} h_l^T \frac{w_l}{\|W\|_w} s_l + n_k \quad \ldots \quad (3.10) \]

3.4 Sum Rate

The achievable sum rate is a quantization method of the system performance. The sum rate follows the Shannon theorem that gives the maximum rate at which transmitter can send the signals over the channel.

This part will contain a description of the sum rate with MF, ZF and MRT, and we will consider a situation at which the total downlink power is fixed and equally shared among all users. So the channel capacity over Additive White Gaussian Noise channel is given [39] by

\[ R = \log_2 (1 + \text{SNR}) \text{(bits/s/Hz)} \quad \ldots \quad (3.11) \]

where

SNR: signal-to-noise ratio.

In Channel state information (CSI) the transmitter transmits multiple data streams to every user concurrently and selectively. All the receivers send the channel estimation feedback to the transmitter on the reverse link, so the transmitter obtains CSI. Hence, the transmitter communicates with all the receivers with perfect CSI. Then, the achievable sum rate per user in a single-cell downlink massive MIMO system, with perfect channel state information is given by
And for K number of users, the achievable sum rate is given as

\[ R_{\text{sum}} = k \log_2(1 + \text{SINR}_k) \] ... (3.13)

3.4.1 Sum Rate for Zero forcing (ZF) precoding

From equation (3.10), the achievable sum rate with ZF can be expressed [40] as

\[ R_{\text{sum}}^{ZF} = k \log_2(1 + \text{SINR}_k^{ZF}) \] ... (3.14)

The sum rate for zero forcing by using vector normalization methods [40] is given by

\[ R_{ZF,\text{vec}} = k \log_2\left(1 + \frac{P_d(M - K + 1)}{K}\right) \] ... (3.15)

The sum rate for zero forcing by using matrix normalization methods [40] is given by

\[ R_{ZF,\text{mat}} = k \log_2\left(1 + \frac{P_d(M - K)}{K}\right) \] ... (3.16)

3.4.2 Sum Rate for Maximum ratio transmission (MRT) precoding

The achievable sum rate with MRT can be written from (3.10) as

\[ R_{\text{sum}}^{MRT} = k \log_2(1 + \text{SINR}_k^{MRT}) \] ... (3.17)

The sum rate for Maximum ratio transmission precoding by using vector normalization methods [32] is given by:

\[ R_{MRT,\text{vec}} \approx k \log_2\left\{1 + \frac{P_d M}{P_d(K - 1) + K}\right\} \] ... (3.18)
The sum rate for Maximum ratio transmission precoding by using matrix normalization methods [32] is given by

\[ R_{\text{MRT}_{\text{mat}}} \approx k \log_2 \left( 1 + \frac{P_d (M + 1)}{P_d (K - 1) + K} \right) \]  \hspace{1cm} \text{(3.19)}

### 3.4.3 Sum Rate for Match Filter (MF) precoding

From equation (3.10), the achievable sum rate with MF can be expressed as

\[ R_{\text{sum}}^{\text{MF}} = k \log_2 (1 + \text{SINR}_{K}^{\text{MF}}) \]  \hspace{1cm} \text{(3.20)}

The sum rate for match filter by using vector/matrix normalization methods [41] is given by

\[ R_{\text{MF}_{\text{vec}}} = R_{\text{MF}_{\text{mat}}} = k \log_2 \left( 1 + \frac{P_d (M + 1)}{P_d (K - 1) + K} \right) \]  \hspace{1cm} \text{(3.21)}

To illustrate the way of applying match filter, zero forcing and maximum ratio transmission linear precoding techniques for single cell downlink massive MIMO system when number of base station antennas from 1 to 100 and from 1 to 200 and number of user from 1 to 50 at two value of power 0dB and -15 dB for matrix and vector normalization methods as shown in the following figures (fig.3.3 and fig.3.4)
Figure 3.3 (a) flow chart of fixed number of antennas with different number of users (b) flow chart of fixed number of users with different number of antennas
CHAPTER FOUR

SIMULATION RESULTS AND DISCUSSIONS

The simulation results compose from six scenarios to compare the performance of MRT, ZF and MF made as follows:

1) Scenario for vector normalization method that increasing the number of antenna with fixing the number of users in high and low downlink transmitting power.

2) Scenario for vector normalization method that fixing the number of antenna with increasing the number of users in high and low downlink transmitting power.

3) Scenario for matrix normalization method that increasing the number of antenna with fixing the number of users in high and low downlink transmitting power.

4) Scenario for matrix normalization method that fixing the number of antenna with increasing the number of users in high and low downlink transmitting power.

5) Scenario for vector normalization versus matrix normalization in high power

6) Scenario for vector normalization versus matrix normalization in low power.

However; the limitation of models are that the number of users $K$ is not greater than the number of antennas $M$ ($M \gg K$). All scenarios simulate by using MATLAB software modelling tools.

4.1 Matrix normalization

We will study the performance of MF, ZF and MRT in single cell downlink massive MIMO system over perfect channel by considering the sum rate depending on Matrix normalization method. Select the number of users $K = 50$ and the number of antennas is 200. Then set up the base station downlink transmitting power to 0 dB & -15 dB.
4.1.1 Performance of MF, ZF and MRT using matrix normalization at K=50 users

Fig: (4.1) illustrates the sum rate across the whole user range according to equations (3.16, 3.19 and 3.21). This scheme consists of the number of antennas \( M = 1:200 \) and the number of users \( K = 50 \). It can be seen from the results that MF and MRT gives best performance at low power and best performance at high value of power when the number of antenna less than 102 antennas. On the other hand, the ZF gives the best performance at high value of power when the number of antenna more than 102 antenna.

![Figure 4.1: Performance of MF, ZF and MRT using matrix normalization [at k=50 & M=1:200].](image)

4.1.2 Performance of MF, ZF and MRT using matrix normalization at M=200 antennas

Figure: (4.2) shows the sum rate across the entire user range according to equations (3.16, 3.19 and 3.21). This scheme consists of the number of antennas \( M = 200 \) and the number of users \( K = 1:50 \). It can be seen from the results that MF and MRT gives the best performance at low value of power when the number of user less than thirty nine users and less than twenty one users at
high power. On the other hand, ZF gives best performance when number of user more than twenty one at high power and more than thirty nine at low power.

Figure 4.2: Performance of MF, ZF and MRT using matrix normalization [at k=1:50 & M=200].

4.2 Vector normalization

This section illustrates the performance of MF, ZF and MRT in single cell downlink massive MIMO system over perfect channel by considering the sum rate depending on vector normalization method. Select the number of users $K = 50$ and the number of antennas is 200. Then set up the base station downlink transmitting power to 0 dB & -15dB.

4.2.1 Performance of MF, ZF and MRT using vector normalization at $K=50$ users

Figure (4.3) shows the sum rate across the entire antenna range according to equations (3.15, 3.18 and 3.21). This scheme consists of the number of antennas $M =1:200$ and the number of users $K = 50$. It can be seen from the result that MF generally gives slightly best performance than other MRT and ZF at low value of power and best performance when number of base station antennas less than 100 at high power. On the other hand, ZF gives best performance at high power when number of base station antennas greater than 100.
Figure 4.3: Performance of MF, ZF and MRT using vector normalization [at k=50 & M=1:200].

4.2.2 Performance of MF, ZF and MRT using vector normalization at M=200 antennas

Fig: (4.4) shows the sum rate across the whole user range according to equations (3.15, 3.18 and 3.21). This scheme consists of the number of antennas M =200 and the number of users K = 1:50. It can be seen from the result that MF generally gives slightly best performance at low value of power. On the other sides, ZF gives best performance at high power.
4.3 Vector normalization versus Matrix normalization

This part shows the comparison of Vector normalization versus Matrix normalization for match filter, zero forcing and maximum ratio transmission. Consider number of users k=50 users, number of antennas equal 100 and 200 and two value of power (0dB & -15dB).

4.3.1 Vector normalization versus Matrix normalization at 200 antennas and 0dB

Figure (4.5) shows the sum rate across the whole antenna range according to equations (3.15 & 3.16 for ZF, 3.18 & 3.19 for MRT and 3.21 for MF). This scheme consists of the number of antennas M =1:200, number of users K = 50 and downlink transmitted power 0dB. It can be seen from the result that vector/matrix normalization for MF and matrix normalization for MRT gives the same performance and gives best performance than ZF and MRT vector normalization when number of antennas less than 100 antennas at high value of power. On the other sides, vector/matrix normalization for ZF is better than MRT vector normalization when the number of antennas greater than 99 antennas and gives less performance when number of antennas greater
than 99 antennas. Also ZF matrix/vector normalization gives best performance than MF matrix/vector normalization and MRT matrix normalization.

![Matrix normalization vs. Vector normalization at k=50, M=1:200 & P_{ad} =0dB.](image)

Figure 4.5: Matrix normalization vs. Vector normalization [at k=50, M=1:200 & P_{ad} =0dB].

4.3.2 Vector normalization versus Matrix normalization at 200 antennas and -15dB

Figure (4.6) shows the sum rate across the whole antenna range according to equations (3.15 & 3.16 for ZF, 3.18 & 3.19 for MRT and 3.21 for MF). This scheme consists of the number of antennas M =1:200, number of users K = 50 and downlink transmitted power -15dB. It can be seen from the result that vector/matrix normalization for MF and MRT matrix normalization gives the same performance and start having noticeable best performance from beginning until the number of antennas reaches 35 it becomes slightly best than the rest normalization methods.
The table 4.1 below gives the sum Rate performance of 1 to 200 numbers of antenna and fixed number of various users 25 and 50 which shows the numerical result of this study. Also table 4.2 illustrates the sum rate performance of 1 to 100 number of antennas and fixed number of different users 250and 50 which shows the numerical result. This numerical result can specify the best performance of three linear precoding techniques in which all techniques gives better performance for different number of antennas and users only they differ in best performance of the rest.
Table 4.1 sum rate improvements (in bps/Hz) for MF, ZF and MRT at M=200 antennas & K=25, 50 users

<table>
<thead>
<tr>
<th>Precoding technique</th>
<th>ZF</th>
<th>MRT</th>
<th>MF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalization method</td>
<td>Vector</td>
<td>Matrix</td>
<td>Vector</td>
</tr>
<tr>
<td>Number of users</td>
<td>25 50</td>
<td>25 50</td>
<td>25 50</td>
</tr>
<tr>
<td>Achievable sum rate at 0dB</td>
<td>75.179 100.389 88.909 128.530</td>
<td>58.632 79.732 58.776 79.973</td>
<td>58.776 79.973</td>
</tr>
</tbody>
</table>

Table 4.2 sum rate improvements (in bps/Hz) for MF, ZF and MRT at M=100 antennas & K=25, 50 users

<table>
<thead>
<tr>
<th>Precoding Technique</th>
<th>ZF</th>
<th>MRT</th>
<th>MF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalization method</td>
<td>Vector</td>
<td>Matrix</td>
<td>Vector</td>
</tr>
<tr>
<td>Number of users</td>
<td>25 50</td>
<td>25 50</td>
<td>25 50</td>
</tr>
<tr>
<td>Achievable sum rate at 0dB</td>
<td>50.358 50.717 63.909 78.53</td>
<td>40.111 50.363 40.352 50.725</td>
<td>40.352 50.725</td>
</tr>
</tbody>
</table>
CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 CONCLUSION

This study investigates the sum rate of three linear precoding techniques: zero forcing, maximum ratio transmission and match filter. The analysis of these precoding depends on matrix normalization and vector normalization at low and high power. The analysis assumes different number of antennas with fixed number of users and different number of users with fixed number of antennas for all scenarios. The result found that in high power, the sum rate improvement for ZF vector normalization and matrix normalization are (100.389 bps/Hz, 128.530 bps/Hz) respectively and in low power (6.579 bps/Hz, 10.471 bps/Hz) respectively, for MRT vector normalization and matrix normalization are (79.973 bps/Hz, 79.732 bps/Hz) respectively and in low power, (8.347 bps/Hz, 8.387 bps/Hz) respectively, for MF vector normalization and MF matrix normalization are equal, in high power (79.973 bps/Hz) and in low power (8.387 bps/Hz). Simulations results show that using linear precoding techniques and increasing number of base station antennas enhance system performance. In conclusion ZF has better performance at vector/matrix normalization in high power. In low power, MF has better performance in vector normalization and ZF has better performance in matrix normalization.

5.2 RECOMMENDATIONS

- Investigate the performance of linear precoding techniques by taking into account the imperfect channel state information (CSI).

- Study the nonlinear precoding techniques for massive MIMO system.

- Compare the performance of the linear and nonlinear precoding techniques for massive MIMO system.

- Study the linear precoding techniques by taking into account pilot contamination phenomena.
REFERENCE


